Modeling the evolution of breast's shape and appearance during radiotherapy

Global Approach and key points

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Overview

Today's journey

Objectives

Main Goal

Follow the evolution of breast shape and volume during **Radiotherapy**.

Related Objectives

1. **Find correspondences** between breast shapes
2. **Model** breast **deformations** using Shape Analysis
3. Suggest a protocol to **optimize dose delivery during therapy**

Context: Radiotherapy

Breast cancer is generally treated in 2 steps:

1. Conservative breast surgery or lumpectomy
2. Breast radiotherapy

Why irradiate after the surgery?

- Insurance to prevent cancer recurrence
- Can treat undetected *in situ* breast cancer

Figure: Breast Radiotherapy image from (Seo et al., 2019)

Context: Radiotherapy

Standard protocol:

Context: Radiotherapy

Standard protocol:

1. Collect patient information: CT scan / breast volume

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1. Collect patient information: CT scan / breast volume
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Acquisition: Each patient undergoes several examinations

Figure: Data Acquisitions for one patient

Problem Approach

Emerging concerns

Breasts can deform between irradiation sessions:

- How to define the ROI?
- Must we change the dose delivery?

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Main steps

1. Solve the **shape matching problem** for the consecutive acquisitions.

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Emerging concerns

Breasts can deform between irradiation sessions:

- How to define the ROI?
- Must we change the dose delivery?

Main steps

1. Solve the **shape matching problem** for the consecutive acquisitions.
2. Use the generated matches to **model the breast deformation** across therapy.

Shape Matching

Objective:

"Given input shapes $S_1, S_2, \ldots, S_N$, establish a meaningful relation between their elements." (van Kaick et al., 2010)

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$\implies$ Very general problem with specific approaches to solve each sub-problems.

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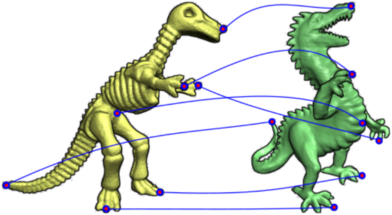

Figure: Sparse Correspondence of features points (van Kaick et al., 2010)

shapes

What is the shape representation?

Shape Matching

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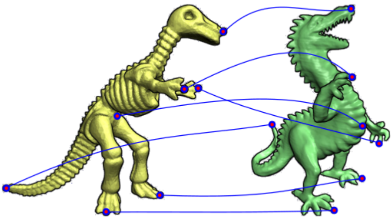

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establish

What approach to find correspondences?

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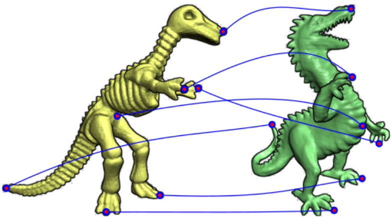

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meaningful

Which correspondence is closer to our goal?

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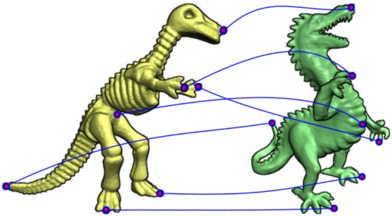


Figure: Sparse Correspondence of features points (van Kaick et al., 2010)

relation

What is the output representation?

What are its properties?

Shape Matching Applications

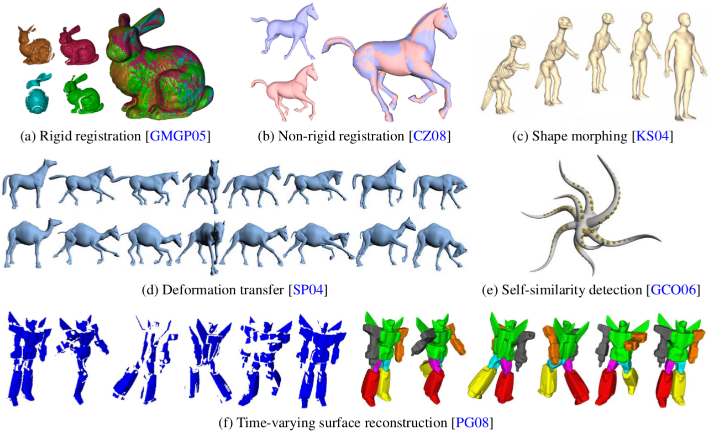

(a) Rigid registration [GMGP05] (b) Non-rigid registration [CZ08] (c) Shape morphing [KS04]
(d) Deformation transfer [SP04] (e) Self-similarity detection [GCO06]
(f) Time-varying surface reconstruction [PG08]

Figure: Possible applications of shape matching (van Kaick et al., 2010)

Many Methods for many problems

Challenging problems

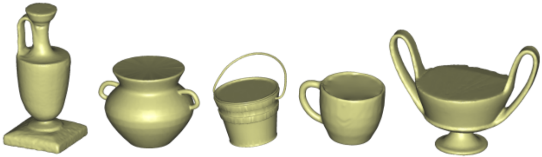


Figure 2: *An example of a collection of man-made shapes (liquid containers) for which computing a correspondence is a challenging problem. Note how the shapes can be constituted by different types and numbers of parts (e.g., one or two handles), how the parts of a same type can vary in their geometry (e.g., long vs. short handles), and how they can connect to each other in different manners.*

Figure: Man-made Shapes (van Kaick et al., 2010)

Many Methods for many problems

Partial Matching

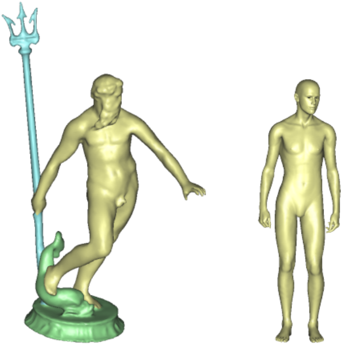


Figure: Partial Matching Example (van Kaick et al., 2010)

And for the thesis?

Challenges

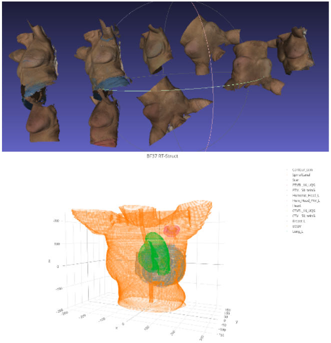
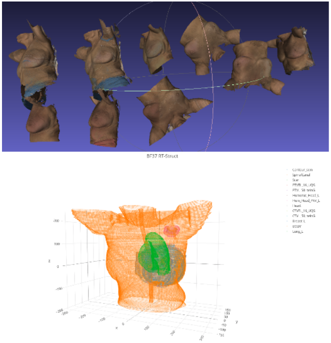


Figure: Patient Acquisitions

And for the thesis?

Challenges

- $\simeq$ Partial Matching between CT acquisitions and textured scans



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- $\simeq$ Partial Matching between CT acquisitions and textured scans
- Non-rigid registration

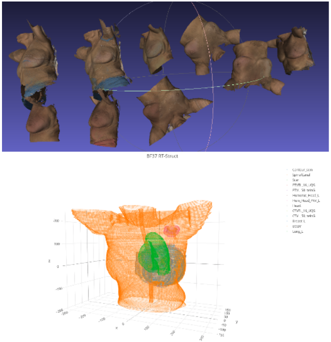
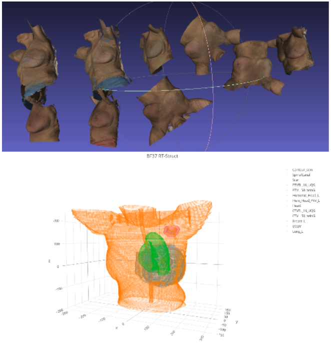


Figure: Patient Acquisitions

And for the thesis?

Challenges

- $\simeq$ Partial Matching between CT acquisitions and textured scans
- Non-rigid registration
- Dense scan acquisitions with $\simeq 300,000$ to $\simeq 1,000,000$ points



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And for the thesis?

Challenges

- $\simeq$ Partial Matching between CT acquisitions and textured scans
- Non-rigid registration
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$\Longrightarrow$ Address the problem with **Functional Maps**

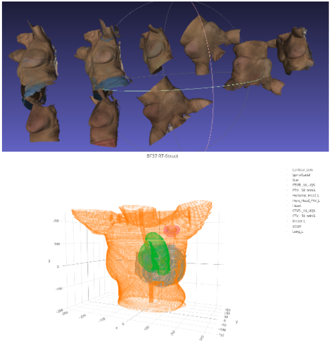

Figure: Patient Acquisitions

Functional Maps

What are Functional Maps (Ovsjanikov et al., 2012) ?

An algebraic formulation of the shape matching problem using *a functional representation of the mapping*.

Mapping

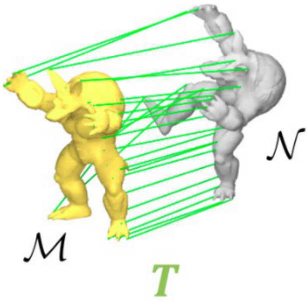

Figure: Point-to-Point mapping T (Ovsjanikov et al., 2017)

Matching Problem:

How to find the mapping?

Resolution of an optimization problem $T_{opt} = \min_T E(T)$

Possible issues

Non-convex / non tractable combinatorial optimization (with multiple minima)

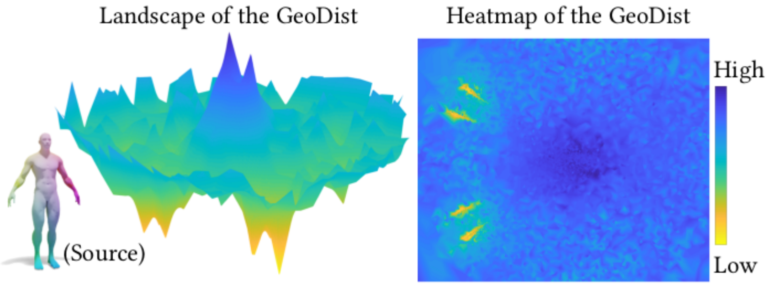

Figure: Geodesic Distortion over 10K self-maps on a human shape (Ren et al., 2020)

Functional Representation

Functional Representation

Use the Dual of the classical point-to-point map T : $T_F : \mathcal{F}(M, \mathbb{R}) \to \mathcal{F}(N, \mathbb{R})$.

$f : M \to \mathbb{R}$ has a transformation $g : N \to \mathbb{R}$ defined by composition $g = f \circ T^{-1}$.

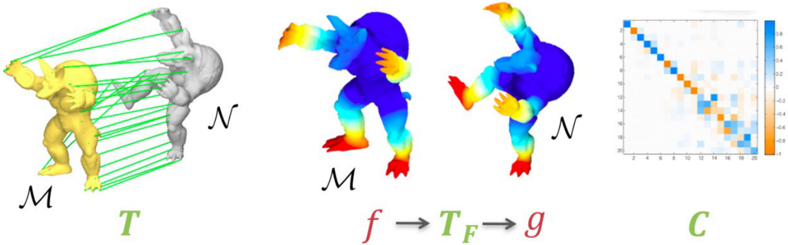

Figure: Ptp map T / Dual map T_F / Corresponding Matrix C (Ovsjanikov et al., 2017)

Functional Representation

Bases for the functional spaces

With bases $\{\phi_i^M\}$ and $\{\phi_i^N\}$ for the function spaces of M and N:

$$f = \sum_{i\geq 1} \underbrace{\langle f, \phi_i^M \rangle_M}_{a_i} \phi_i^M \text{ and } g = \sum_{i\geq 1} \underbrace{\langle g, \phi_i^N \rangle_N}_{b_i} \phi_i^N$$

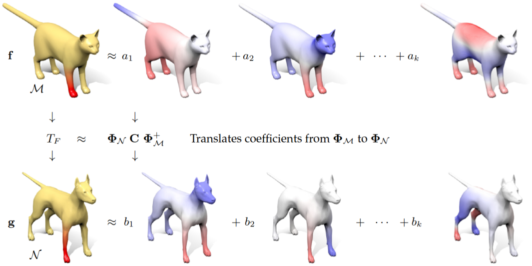

Figure: Illustration showing how the map is encoded by the matrix C (Ovsjanikov et al., 2017)

A physic intuition

The role of modes and frequencies:

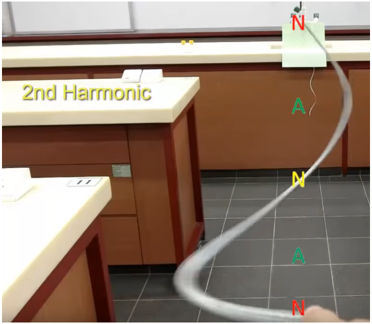

Figure: Rope second harmonic

The wave equation is

- widely used to describe the propagation of oscillations about an equilibrium
- given by: $\Delta f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$
- solved by separating time and space: $f(x, y, t) = \phi(x, y)h(t)$ $\implies \frac{\Delta\phi}{\phi} = \frac{h''}{h} = \lambda$

Finally, by solving an **eigenproblem** $\Delta\phi = \lambda\phi$, we can find **stationary waves**.

Closer to a geometric interpretation

Why do we want oscillation frequencies λ and stationary waves ϕ?

- $\rightarrow$ frequencies λ are conditioned by the rope length
- $\rightarrow$ solutions ϕ describe possibles behaviors of the rope
- $\rightarrow$ stationary waves contains **nodes** (points in 1D) where the oscillation amplitude is null

And in 2 or 3 dimensions?

Nodes are **lines and curves in 2D** and **2D planes in 3D**.

$\Longrightarrow$ Can those **spectral** quantities describe surfaces ?

Spectral representation of shapes

Direct and Inverse Problems

1. Given a shape S, can we deduce something about it spectrum?
2. Conversely, given a spectrum, what can we learn about the shape? or "Can we hear the shape of a drum?"

$\Longrightarrow$ Study of **Spectral properties of shapes** to solve various problems:

- Shape matching
- Shape analysis
- Shape retrieval/ recovery from the spectrum

Let's focus on the first one!

Functional Representation

Formulation of the shape matching problem

If f and g corresponds, we must have $T_F(f) = g$.

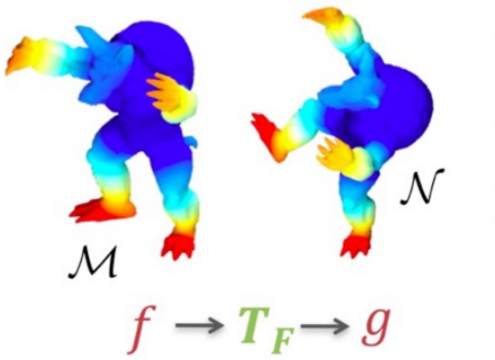

Figure: Functional Correspondence (Ovsjanikov et al., 2017)

Functional Representation

Formulation of the shape matching problem

If f and g corresponds, we must have $T_F(f) = g$.

$$T_F(f) = T_F\left(\sum_i a_i \phi_i^M\right) = \sum_i a_i T_F(\phi_i^M)$$
$$= \sum_i a_i \sum_j \underbrace{\langle T_F(\phi_i^M), \phi_j^N \rangle_N}_{c_{j,i}} \phi_j^N = \sum_j \sum_i a_i c_{j,i} \phi_j^N \quad (1)$$

and

$$g = \sum_j b_j \phi_j^N \quad (2)$$

Functional Representation

Using equations (1) and (2), the correspondence between f and g is written $Ca = b$.

Estimation of C, the mapping matrix

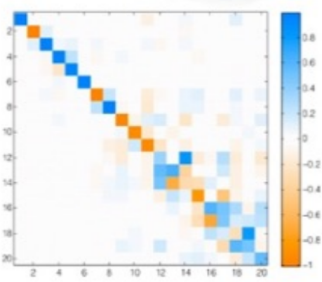

Figure: Correspondence Matrix

Functional Representation

Using equations (1) and (2), the correspondence between f and g is written $Ca = b$.

Estimation of C, the mapping matrix

We expect $f : M \to \mathbb{R}$ and $g : N \to \mathbb{R}$ to correspond (texture, curvature, etc) $\to$ C must satisfy $Ca \simeq b$.

$\implies$ Given enough pairs $\{a_j, b_j\}$, C is found by solving a linear system $CA = B$ in the least square sense where a_j and b_j are columns of A and B.

Functional Maps Computation

Preservation of function constraints

- Descriptor preservation
  - Wave Kernel Signature (Aubry et al., 2011)
  - Heat Kernel Signature (Sun et al., 2009)
  - Gaussian Curvature
  - SHOT (Tombari et al., 2010)
- Texture preservation
- Landmark/Part correspondences

$\Longrightarrow$ Minimize $E_{desc}(C) = ||CA - B||^2$

Functional Maps Computation

Operator Commutativity

Preservation of linear functional operators on M and N (**Symmetry operator**, Laplace-Beltrami operator).

We want C to commute with particular operators:

Let S_F^M and S_F^N be functional operators on M and N, we want

$$S_F^N \circ T_F = T_F \circ S_F^M .$$

In matrix notation: $\|S_F^N C - C S_F^M\| = 0$.

$\implies$ Minimize $E_{comm}(C) = \|S_F^N C - C S_F^M\|^2$

Functional Maps Computation

Functional Map Estimation

Resolve the following optimization problem:

$$C_{opt} = \underset{C}{\operatorname{argmin}} \left(E_{desc}(C) + E_{comm}(C) \right).$$

Functional Maps Computation

Functional Map Estimation

Resolve the following optimization problem:

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We can add regularization constraints:

- If the map T is **volume preserving**, its matrix C must be **orthonormal** i.e $C^T C = I. \implies E_{ortho}(C) = \|C^T C - I\|^2$
- If T is an **isometry**, the matrix C commutes with the Laplace-Beltrami Operator. $\implies E_{iso}(C) = \|C\Lambda^M - \Lambda^N C\|^2$ with Λ^M, Λ^N the diagonal matrices of eigenvalues of M and N Laplacian operators.

Functional Maps Computation

Conversion to Point-to-Point Map

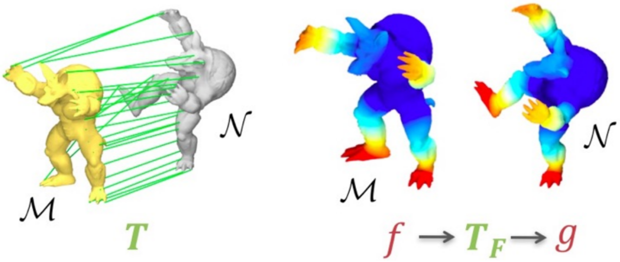

Figure: PtP/Functional Mappings

Functional Maps Computation

Conversion to Point-to-Point Map

Use of indicator functions of highly peaked Gaussian:

1. $f = \delta_x$ for a point $x \in M$
2. compute $T_F(\delta_x)$ and find the closest function $g = \delta_y$ on N

Using the Laplace-Beltrami basis, one can use an efficient procedure to do it on all points at once.

Functional Maps Computation

Conversion to Point-to-Point Map

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2. compute $T_F(\delta_x)$ and find the closest function $g = \delta_y$ on N

Using the Laplace-Beltrami basis, one can use an efficient procedure to do it on all points at once.

Remark: Thanks to the Plancherel's theorem, given $g_1, g_2 \in \mathcal{F}(N, \mathbb{R})$, with spectral coefficients $\mathbf{s}_1$ and $\mathbf{s}_2$, we have:

$$\sum_i (s_{1i} - s_{2i})^2 = \int_N (g_1(y) - g_2(y))^2 \mu(y).$$

Functional Maps Computation

Post-Processing Iterative Refinement

Improve a generated map using an ICP like technique on the embedded functional space $\rightarrow$ ZoomOut (Melzi et al., 2019).

Functional Maps Computation

Post-Processing Iterative Refinement

Improve a generated map using an ICP like technique on the embedded functional space → ZoomOut (Melzi et al., 2019).

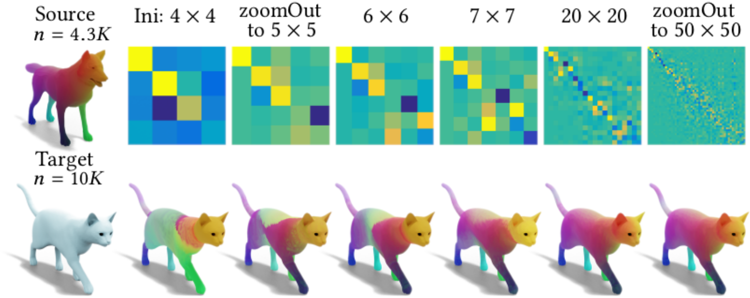

Figure: Exemple of ZoomOut Refinement (Melzi et al., 2019)

Functional Maps conclusion

The functional representation of the mapping

- generalizes the Point-to-Point mapping representation.
- allows to use various descriptors.
- allows many constraints to be linear. $\rightarrow$ **efficient inference**
- implies maps can be easily manipulated via algebraic operations (addition, composition, etc).

$\Longrightarrow$ allows the use of flexible methods.

But also

- seems to be sensitive to noisy data.
- involves a dependence on the chosen descriptors.

Functional Maps conclusion

A lot of methods developed with functional maps

- ZoomOut (Melzi et al., 2019)
- MapTree (Ren et al., 2020)
- Fully Spectral Partial Shape Matching (Litany et al., 2017)
- Partial Functional Correspondence (Rodolà et al., 2015)
- Functional Maps (Ovsjanikov et al., 2012)

Always trying to use LBO as an intrinsic descriptor and to reduce limitations like the descriptor choice

Functional Maps conclusion

Leading to very interesting new methods that are efficient in many complex cases:

- Instant recovery of shape from spectrum via latent space connections (Marin et al., 2020)
- Universal Spectral Adversarial Attacks for Deformable Shapes (Rampini et al., 2021)
- Spectral Unions of Partial Deformable 3D Shapes (Moschella et al., 2021)
- Wavelet-based Heat Kernel Derivatives: Towards Informative Localized Shape Analysis (Kirgo et al., 2020)
- Orthogonalized Fourier Polynomials for Signal Approximation and Transfer (Eurographics 2021)
- A parametric analysis of discrete Hamiltonian functional maps (Postolache et al., 2020)
- LIMP: Learning Latent Shape Representations with Metric Preservation Priors (Cosmo et al., 2020)

Conclusion

My thesis focus

- Implement a Partial Matching strategy to match CT point clouds and textured meshes

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Future Steps

- Work on CT RT-Structures (segmentations) to define breast region using lead wire/CTV.
- Align surfaces for a better visualization.
- Use Latent Space Shape Difference (LSSD) Operators to model and follow deformations across radiotherapy.
- Transform Slices Dose Maps to 3D Point clouds/Meshes to search for correlations with displacements.

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Data

ICANS Clinical trials

Inclusion/Exclusion criteria such as Body Mass Index (BMI) $\simeq$ 30 or breasts with a C cup size to avoid difficult data.

As a result, we have at our disposal for each of the 60 patients:

- $\simeq$ 10 meshes of the front part of the torso
- 1 CT scan containing a point cloud representation of
  - the from shoulder to hips surface skin contour
  - a lead wire around the treated breast
  - other structures like the breast, the heart etc

$\Longrightarrow$ $\simeq$ 600 surface scans and 60 CT scans (18 available as of now)

Data

DICOM data

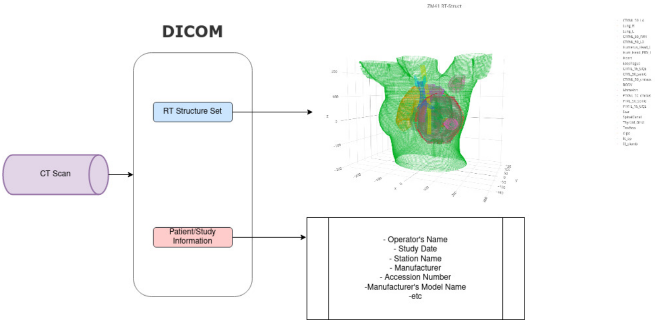

Figure: CT scan data structure

Data

Surface data

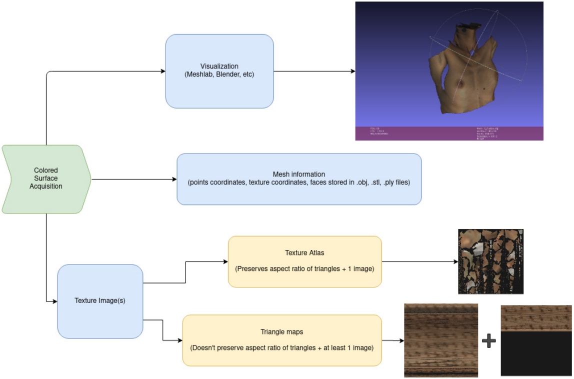

Figure: Surface acquisition data structure

Data

For one patient we have:

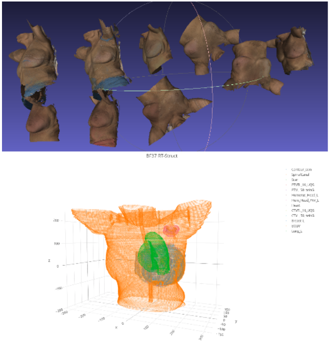

Figure: Surface acquisition and RT-Struct for patient BF37

Data

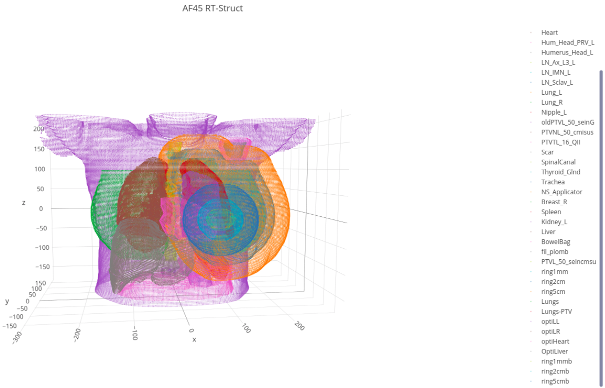

Figure: Example of DICOM RT-Struct with radiation information

Data

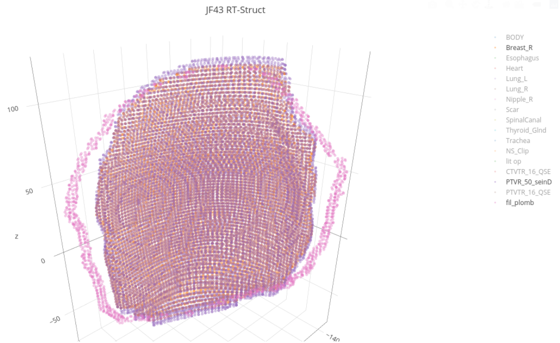

Figure: Example of DICOM RT-Struct with only breast information